Interference Estimation and Mitigation for STAP using the Two-Dimensional Wold Decomposition Parametric Model

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Overview

We propose a new approach for parametric modeling and estimation of STAP data, based on the 2-D Wold like decomposition of random fields.

- The proposed parametric estimation algorithms of the interference components simplify and improve existing STAP methods.

- The resulting modeling and processing methods provide new parametric tools to estimate and mitigate the Doppler ambiguous clutter.

- The estimation algorithms we propose enable the estimation of the interference signals using the observations in *only* a single range gate.

- The proposed method is particularly suitable for non-stationary clutter and jamming environments.

- The approach also provides a new analytical insight into the STAP problem.

The STAP Problem

- The goal of space-time adaptive processing is to manipulate the available data to achieve high gain at the target angle and Doppler and maximal mitigation along both the jamming and clutter lines.

- Because the interference covariance matrix is unknown a priori, it is typically estimated using sample covariances obtained from averaging over a few range gates.

The Approach

- We adopt the 2-D Wold like decomposition of random fields as the parametric model of the observed data.
- Employing this model, we derive computationally efficient algorithms useful for parametrically estimating both the jamming and clutter fields.
- Having estimated the interference terms parametric models, their covariance matrix can be evaluated based on the estimated parameters.
- Once the parametric models of the interference components have been estimated, several alternatives for mitigating the interference are available.
  - Parametric fully adaptive processing.
  - Parametric partially adaptive processing – only the spectral support parameters of the interference components need to be estimated.

The 2-D Wold-Like Decomposition

Theorem: Let $\{y(n,m),\ (n,m) \in \mathcal{Z}^2\}$ be a regular random field. Then $\{y(n,m)\}$ can be represented **uniquely** by the **orthogonal decomposition**:

$$y(n,m) = w(n,m) + v(n,m) \ ,$$

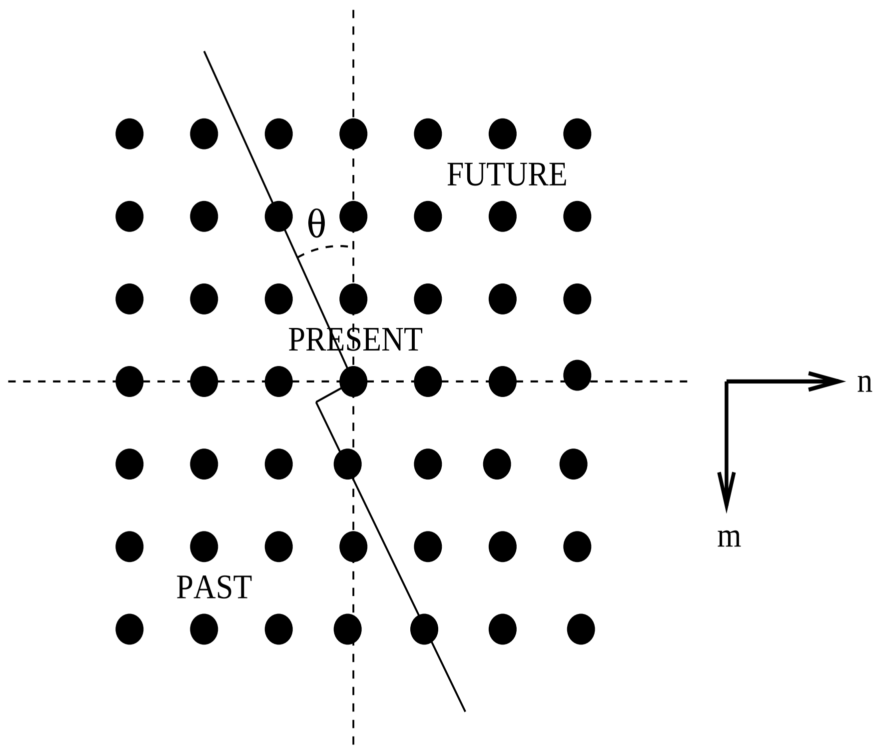
$$w(n,m) = \sum_{(0,0) \preceq (k,l)} a(k,l) u(n-k, m-l) \ ,$$

$u(n,m)$ is the white innovations field of $y(n,m)$.

The field $\{w(n,m)\}$ is purely-indeterministic and regular.

The field $\{v(n,m)\}$ is deterministic.

The Prediction Support



RNSHP support; example with $\alpha = 2$ and $\beta = 1$.

The Deterministic Component Decomposition

$$v(n,m) = h(n,m) + \sum_{(\alpha,\beta)\in O} e_{(\alpha,\beta)}(n,m)$$

where $\{h(n,m)\}$ is a **harmonic** random field and $\{e_{(\alpha,\beta)}(n,m)\}$ is an **evanescent** field .

O is a set of pairs (α,β) of coprime integers.

All decomposition components are mutually orthogonal.

Modeling the Evanescent Field

For any $(\alpha, \beta) \in O$

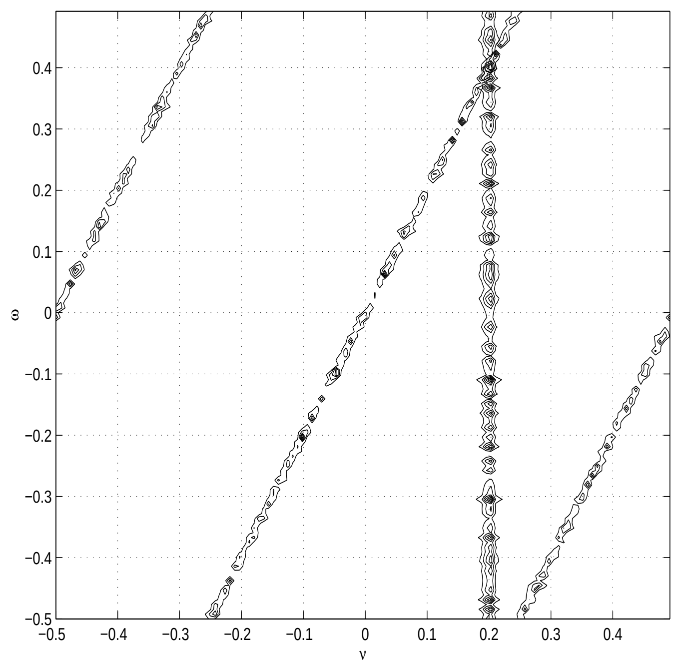
$$e_{(\alpha,\beta)}(n,m) =$$

$$\sum_i s_i^{(\alpha,\beta)}(n\alpha - m\beta) e^{j2\pi \frac{\nu_i^{(\alpha,\beta)}}{\alpha^2+\beta^2}(n\beta + m\alpha)} .$$

The 1-D purely-indeterministic, complex valued processes $\{s_i^{(\alpha,\beta)}(n\alpha - m\beta)\}$ and $\{s_j^{(\alpha,\beta)}(n\alpha - m\beta)\}$, are zero-mean and mutually orthogonal for all $i \neq j$.

The "spectral density function" of each evanescent field has the form of a sum of 1-D delta functions which are supported on lines of rational slope in the 2-D spectral domain.

The Spectrum of Evanescent Fields – Example



DFT of an observed field with two evanescent components.

The Harmonic Component

$$h(n,m) = \sum_p C_p e^{j2\pi(n\omega_p + m\nu_p)} \ .$$

The C_p 's are mutually orthogonal random variables, $E|C_p|^2 = \sigma_p^2$, and (ω_p, ν_p) are the spatial frequencies of the $p-th$ harmonic.

STAP and the 2-D Wold Decomposition

- The target signal is modeled as a random amplitude complex exponential – in the space-time domain the target model is that of a 2-D harmonic component.

- The purely-indeterministic component of the space-time field is the sum of a white noise field due to the internally generated receiver amplifier noise, and a colored noise field due to the sky noise contribution.

- The interference components (jammers and clutter) are evanescent random fields:

  - In the space-time domain each jammer is modeled as an evanescent component with $(\alpha, \beta) = (1, 0)$ whose 1-D modulating process is white noise.

  - Clutter from all angles lies in a "clutter ridge", supported on a diagonal line (that generally wraps around in Doppler) in the angle-Doppler domain, with (α, β) such that $\tan \beta/\alpha$ corresponds to the slope of the clutter ridge.

Setting the Estimation Problem

Let $\{y(n,m)\}$, $(n,m) \in D$ where $D = \{(i,j) | 0 \leq i \leq S-1, 0 \leq j \leq T-1\}$ be the observed random field.

Define

$$y = [y(0,0), \ldots, y(0,T-1), y(1,0), \ldots, y(1,T-1), \ldots, \\ \ldots, y(S-1,0), \ldots, y(S-1,T-1)]^T$$

$$e_i^{(\alpha,\beta)} = [e_i^{(\alpha,\beta)}(0,0), \ldots, e_i^{(\alpha,\beta)}(0,T-1), e_i^{(\alpha,\beta)}(1,0) \\ , \ldots, e_i^{(\alpha,\beta)}(1,T-1), \ldots, \ \ldots, e_i^{(\alpha,\beta)}(S-1,0), \\ \ldots, e_i^{(\alpha,\beta)}(S-1,T-1)]^T$$

Setting the Estimation Problem

Let

$$\begin{aligned}\tilde{s}_i^{(\alpha,\beta)} = [&s_i^{(\alpha,\beta)}(0), s_i^{(\alpha,\beta)}(-\beta), \ldots, s_i^{(\alpha,\beta)}(-(T-1)\beta),\\ &s_i^{(\alpha,\beta)}(\alpha), s_i^{(\alpha,\beta)}(\alpha-\beta) \ldots, s_i^{(\alpha,\beta)}(\alpha-(T-1)\beta) \ldots\\ &s_i^{(\alpha,\beta)}((S-1)\alpha) \ldots, s_i^{(\alpha,\beta)}((S-1)\alpha-(T-1)\beta)]^T\end{aligned}$$

be the vector of the observed samples from the 1-D modulating process $\{s_i^{(\alpha,\beta)}\}$ of the evanescent field $\{e_i^{(\alpha,\beta)}\}$.

Define

$$\begin{aligned}v^{(\alpha,\beta)} = [\ &0,\ \alpha,\ \ldots,\ (T-1)\alpha,\\ &\beta,\ \beta+\alpha,\ \ldots,\ \beta+(T-1)\alpha,\ \ldots,\ldots,\\ &(S-1)\beta,\ (S-1)\beta+\alpha, \ldots\ (S-1)\beta+(T-1)\alpha]^T\end{aligned}$$

Then

$$d_i^{(\alpha,\beta)} = \exp\left(j2\pi \tfrac{\nu_i^{(\alpha,\beta)}}{\alpha^2+\beta^2} v^{(\alpha,\beta)}\right)$$

$$e_i^{(\alpha,\beta)} = \tilde{s}_i^{(\alpha,\beta)} \odot d_i^{(\alpha,\beta)}\ .$$

Properties of the Evanescent Field

Whenever $n\alpha - m\beta = k\alpha - \ell\beta$ for some integers n, m, k, ℓ such that $0 \leq n, k \leq S - 1$ and $0 \leq m, \ell \leq T - 1$, the same element of $\tilde{s}_i^{(\alpha,\beta)}$ appears more than once in the vector.

Lemma: For a rectangular observed field of dimensions $S \times T$ the number of *distinct* samples from the random process $\{s_i^{(\alpha,\beta)}\}$ that are found in the observed field is $(S-1)|\alpha| + (T-1)|\beta| + 1 - (|\alpha| - 1)(|\beta| - 1)$

The *concentrated version*, $s_i^{(\alpha,\beta)}$ of $\tilde{s}_i^{(\alpha,\beta)}$ is a $(S-1)|\alpha| + (T-1)|\beta| + 1 - (|\alpha| - 1)(|\beta| - 1)$ column vector of non-repeating samples of the process $\{s_i^{(\alpha,\beta)}\}$.

$$\tilde{s}_i^{(\alpha,\beta)} = A_i^{(\alpha,\beta)} s_i^{(\alpha,\beta)}$$

where $A_i^{(\alpha,\beta)}$ is rectangular matrix of zeros and ones which replicates rows of $s_i^{(\alpha,\beta)}$.

The Covariance Matrix of the Field

The covariance matrix $R_i^{(\alpha,\beta)}$ which characterizes $\{s_i^{(\alpha,\beta)}\}$ is defined in terms of the concentrated version vector

$$R_i^{(\alpha,\beta)} = E[s_i^{(\alpha,\beta)}(s_i^{(\alpha,\beta)})^H] \ .$$

The covariance matrix of the evanescent component is given by

$$\Gamma_i^{(\alpha,\beta)} = (A_i^{(\alpha,\beta)} R_i^{(\alpha,\beta)} (A_i^{(\alpha,\beta)})^T) \odot (d_i^{(\alpha,\beta)}(d_i^{(\alpha,\beta)})^H) \ .$$

Since the evanescent components $\{e_i^{(\alpha,\beta)}\}$, are mutually orthogonal, and since all the evanescent components are orthogonal to the purely-indeterministic component

$$\Gamma = \Gamma_{PI} + \sum_{(\alpha,\beta)\in O} \sum_{i=1}^{I^{(\alpha,\beta)}} \Gamma_i^{(\alpha,\beta)} \ .$$

Estimating the Interference Parametric Model

- Estimate the (α, β) pair.
- For a fixed $c = n\alpha - m\beta$ (i.e., along a line on the sampling grid), the samples of the evanescent component are the samples of 1-D constant amplitude harmonic signal, whose frequency is $\nu_i^{(\alpha,\beta)}$.

$$\hat{s}_i^{(\alpha,\beta)}(c) = \frac{1}{N_s} \sum_{n\hat{\alpha} - m\hat{\beta} = c} y(n,m) \exp(-j2\pi \frac{\hat{\nu}_i^{(\alpha,\beta)}}{\hat{\alpha}^2 + \hat{\beta}^2}(n\hat{\beta} + m\hat{\alpha}$$

N_s denotes the number of the observed field samples that satisfy the relation $n\alpha - m\beta = c$.

- Estimate the parameters of the 1-D AR (or any other) model of $s_i^{(\alpha,\beta)}$.

Fully Adaptive Parametric Processing

Having estimated the parametric models of the noise and interference components of the field, the estimated parameters are substituted into the parametric expression of the covariance matrix to obtain an estimate of the interference-plus-noise covariance matrix Γ.

Weights in the optimal space-time filter are given by

$$w = \Gamma^{-1} v_t$$

The test statistic is

$$z(\varpi, \vartheta) = w^H(\varpi, \vartheta) y$$

Partially Adaptive Parametric Processing

After demodulation using the estimated $\alpha, \beta, \nu_i^{(\alpha,\beta)}$

$$\Gamma_i^{(\alpha,\beta)} = A_i^{(\alpha,\beta)} R_i^{(\alpha,\beta)} (A_i^{(\alpha,\beta)})^T \ .$$

Construct the following orthogonal projection matrix

$$T_i^{(\alpha,\beta)} = A_i^{(\alpha,\beta)}((A_i^{(\alpha,\beta)})^T A_i^{(\alpha,\beta)})^{-1}(A_i^{(\alpha,\beta)})^T \ .$$

It is easily verified (by substitution) that $T_i^{(\alpha,\beta)}$ is an orthogonal projection onto the range space of $\Gamma_i^{(\alpha,\beta)}$ since

$$\Gamma_i^{(\alpha,\beta)} v = \Gamma_i^{(\alpha,\beta)} T_i^{(\alpha,\beta)} v \ .$$

The projection matrix onto the subspace orthogonal to the clutter space is given by $(T_i^{(\alpha,\beta)})^{\perp} = I - T_i^{(\alpha,\beta)}$.

- $A_i^{(\alpha,\beta)}$ is a sparse matrix of zeros and ones, whose structure is a function of α and β *only*. Hence, $\mathrm{T}_i^{(\alpha,\beta)}$ is easily computed.

- *Only* $\alpha, \beta, \nu_i^{(\alpha,\beta)}$ need to be estimated !